STUDY GUIDE

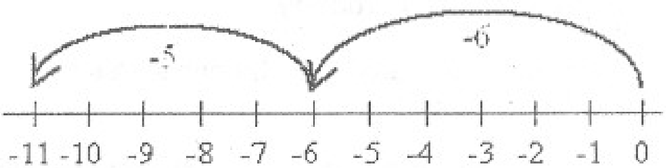
Add & Subtract Integers Fact Sheet

Adding negative integers.

(−1) + (−3) = (−4)

(−6) + (−12) + (−3) + (−10) + (−5) = −36
debt debt debt debt debt = lots of debt

You jump on the number line towards the negative (left), more and more.



(−6) + (−5) = −11

Just add the absolute values, and put the negative sign in front.

Subtracting a positive integer.

2 − 1 = 1	(−4) − 0 = −4
2 − 2 = 0	(−4) − 1 = −5
2 − 3 = −1	(−4) − 2 = −6
2 − 4 = −2	(−4) − 3 = −7
etc.	etc.

It is like temperature dropping, or money being subtracted from a bank account. Subtracting a positive integer just means more debt.

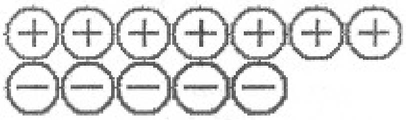
On a number line, subtracting 7 means jumping 7 steps towards the left.

5 − 8 is also the same as 5 + (−8). In other words, you can change subtracting a number into addition of the opposite number.

Let's say the answer to (−2) − 6 is A and we don't know it yet. Since subtraction is the opposite operation of addition, A + 6 = −2. A = −8 is the only number that works.

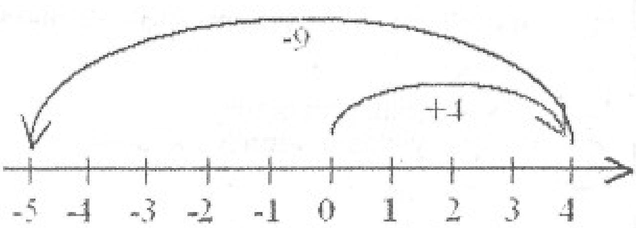
Adding integers with different signs.

7 + (−5) = ___



Some of the positives & negatives cancel each other. The difference of the absolute values tells you how many didn't get canceled.

Jump on the number line: a positive number is a jump to the right, a negative number means a jump to the left.



4 + (−9) = −5.

Subtracting a negative integer.

2 − 2 = 0	(−7) − 2 = (−9)
2 − 1 = 1	(−7) − 1 = (−8)
2 − 0 = 2	(−7) − 0 = (−7)
2 − (−1) = 3	(−7) − (−1) = (−6)
2 − (−2) = 4	(−7) − (−2) = (−5)
etc.	etc.

When you subtract a negative integer, change it into adding the opposite of the negative number, which is of course positive:

8 − (−4) = 8 + 4 = 12.
(−10) − (−4) = (−10) + 4 = (−6)

Two negatives changes into one positive!

On number line, subtracting (−4) means jumping 4 steps to the left - but before you jump, the extra minus makes you turn around, so you jump to the right instead.

Let's say the answer to 2 − (−3) is B and we don't know it yet. Since subtraction is the opposite operation of addition, B + (−3) = 2. B = 5 is the only number that works.

Similarly, if (−7) − (−3) is C, then C + (−3) = −7. So C is (−4).

Multiply & Divide Integers Fact Sheet

A positive integer times a negative integer:

Think of repeated addition here: $3 \times (-2) =$

⊖⊖ and ⊖⊖ and ⊖⊖

$(-2) \quad + \quad (-2) \quad + \quad (-2) \quad = -6.$

Or, $4 \times (-7) = (-7) + (-7) + (-7) + (-7) = -28.$

A positive integer times a negative integer:

Since you can change the order of the factors,

$(-6) \times 4 = 4 \times (-6) = -24.$

In general, if m and n are natural numbers, then $m \times (-n)$ is $(-n)$ added repeatedly m times, so is negative. And $(-m) \times n$ is the same as $n \times (-m)$ and so is negative as well.

⊖ × ⊕
⊕ × ⊖ both have a negative answer

Dividing a negative integer by a positive.

⊖⊖⊖
⊖⊖⊖

Divide these negatives into three equal groups.

$(-6) \div 3 = -2.$

Dividing a positive integer by a negative.

What is $(-15) \div 5$? Let's call the answer Z. Since division and multiplication are opposite operations, $Z \times 5 = -15$. So Z must be -3.

In general, if m and n are natural numbers, and $(-m) \div n$ is B, then $B \times n = (-m)$, and B must be negative.

Dividing a negative integer by a negative.

Let's say $(-21) \div (-7)$ is some number A.

It follows that $A \times (-7) = (-21)$

Knowing the multiplication rules,
the only number that fits A is 3.

In general, if m and n are natural numbers, and $(-m) \div (-n)$ is B, then $B \times (-n) = (-m)$, and B must be positive.

A negative times a negative.

$(-3) \times 3 =$
$(-3) \times 2 =$
$(-3) \times 1 =$
$(-3) \times 0 =$
$(-3) \times (-1) =$
$(-3) \times (-2) =$
$(-3) \times (-3) =$
$(-3) \times (-4) =$

Complete the pattern on the left. Observe how the products continually increase by 3 in each step.

It follows that the *negative times negative* products in the pattern must be positive.

Another 'justification' for this rule can be seen using distributive property:

Distributive property of arithmetic states that $a(b + c) = ab + ac$.

So, if $a = (-1)$, $b = 3$, and $c = (-3)$, it should still hold:

$(-1)(3 + (-3)) = (-1)(3) + (-1)(-3)$

Now, since $3 + (-3)$ is zero, the whole left side is zero. So $(-1)(3) + (-1)(-3)$ must be zero as well.

$(-1)(3)$ is -3. So it follows that $(-1)(-3)$ has to be the opposite of -3, or 3.

The *'negative times negative makes positive'* rule has to do with the fact that IF we made it to be negative, then all these neat rules and properties of arithmetic wouldn't hold for negative numbers.

But mathematicians do want them to hold, since we DO want mathematics to be a very consistent system. So the convention is made that negative times negative is positive.

In a nutshell, whether you multiply *or* divide:

⊕ ×/÷ ⊖
⊖ ×/÷ ⊕
(different signs) yields a negative answer

⊕ ×/÷ ⊕
⊖ ×/÷ ⊖
(same kind of signs) yields a positive answer